

AD-755 404

INVESTIGATION OF LASER PROPAGATION
PHENOMENA

Stuart A. Collins, Jr., et al

Ohio State University

Prepared for:

Rome Air Development Center
Advanced Research Projects Agency

November 1972

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

RADC-TR-72-324
Technical Report
November 1972



INVESTIGATION OF LASER PROPAGATION PHENOMENA (432-3)

The Ohio State University ElectroScience Laboratory

Department of Electrical Engineering
Columbus, Ohio 43210

Sponsored by
Defense Advanced Research Projects Agency
DAPA Order No. 1379

Approved for public release;
distribution unlimited.



The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Defense Advanced Research Projects Agency or the U. S. Government.

Prescribed by
NATIONAL TECHNICAL
INFORMATION SERVICE
Department of Commerce
Springfield, VA 22151

Form Air Development Center
Air Force Systems Command
Griffiss Air Force Base, New York

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

The Ohio State University ElectroScience Laboratory
Department of Electrical Engineering
Columbus, Ohio

20. REPORT SECURITY CLASSIFICATION

Unclassified

20a. GROUP

3. REPORT TITLE

INVESTIGATION OF LASER PROPAGATION PHENOMENA

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Quarterly Technical Report

5. AUTHOR(S) (Last name, first name, initial)

Collins, S.A., Jr.
Reinhardt, G.W.

6. REPORT DATE

7a. TOTAL NO. OF PAGES

19

7b. NO. OF REFS

6

8a. CONTRACT OR GRANT NO.

F30602-72-C-0305

9a. ORIGINATOR'S REPORT NUMBER(S)

ElectroScience Laboratory 3432-3

b. PROJECT NO.

c. TASK

d.

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

RADC-TR-72-328

10. AVAILABILITY/LIMITATION NOTICES

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

Defense Advanced Research
Projects Agency
Washington, D.C.

13. ABSTRACT

A complete analytical description of a device to measure angle of arrival is presented along with predictions of expected results for various input apertures and turbulence conditions. This work is necessary for defining experiments made with the instrument of RADC. Also presented are some extensions of previous work on temporal spectra of phase difference measurements. Specifically the case for slanted paths is considered.

14.

KEY WORDS

LINK A

LINK B

LINK C

ROLE

WT

ROLE

WT

ROLE

WT

Angle of arrival
Phase fluctuations
Propagation
Refraction index effects
Measurements
Temporal spectra
Phase difference
Slant paths

INSTRUCTIONS

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parentheses immediately following the title.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

INVESTIGATION OF LASER PROPAGATION PHENOMENA

S. A. Collins, Jr.
G. W. Reinhardt

Contractor: Ohio State University
Contract Number: F30602-72-C-0305
Effective Date of Contract: 1 April 1972
Contract Expiration Date: 31 March 1973
Amount of Contract: \$75,000.00
Program Code Number: 9E20

Principal Investigator: Dr. Stuart A. Collins, Jr.
Phone: 614 422-5045

Project Engineer: Edward K. Damon
Phone: 614 422-5953

Contract Engineer: Raymond P. Urtz, Jr.
Phone: 315 330-3443

Approved for public release;
distribution unlimited.

This research was supported by the
Advanced Research Projects Agency
of the Department of Defense and
was monitored by Raymond P. Urtz,
Jr. RADC (OCSE), GAFB, NY 13441
under Contract F30602-72-C-0305.

PUBLICATION REVIEW

This technical report has been reviewed and is approved


Raymond P. Artz Jr.
RADC Project Engineer

ABSTRACT

A complete analytical description of a device to measure angle of arrival is presented along with predictions of expected results for various input apertures and turbulence conditions. This work is necessary for defining experiments made with the instrument of RADC. Also presented are some extensions of previous work on temporal spectra of phase difference measurements. Specifically the case for slanted paths is considered.

CONTENTS

	Page
INTRODUCTION	1
ANGLE OF ARRIVAL APPARATUS	1
PHASE DIFFERENCE TEMPORAL SPECTRA ALONG SLANT PATHS	14
SUMMARY AND CONCLUSIONS	18
BIBLIOGRAPHY	19

INTRODUCTION

This is the second quarterly technical report under Contract F30602-72-C-0305. The object of the contract is to provide theoretical support to the experimental program in atmospheric propagation of 10.6 micron light beams at the Rome Air Development Center. Specific problems such as theoretical prediction of temporal spectra of light beam amplitude and phase fluctuations, averaging time required for optical atmospheric data have been the subjects of past investigations. The work to be reported on this quarter is a theoretical investigation of the apparatus used for the measurement of angle of arrival at RADC. A complete analytical description of device operation is presented along with predictions of expected results for various input apertures and turbulence conditions. This work is necessary for defining experiments and interpreting results of measurements made with the instrument.

Also presented are some extensions of previous work on temporal spectra of phase difference measurements. Specifically the case for slanted paths is considered. This work applies to measurements made when the optical receiver is in the receiving tower at the RADC PATS site, and for vertical beams.

The angle of arrival apparatus will be described first.

ANGLE OF ARRIVAL APPARATUS

The following is a description of the apparatus used at RADC to measure angle of arrival. The description was started in conjunction with a report on angle of arrival (3163-1) put out under a previous contract. The description was put aside to be completed at a later date, a task which is performed herein.

The description was undertaken to provide a better understanding of an arrival angle meaning device designed and constructed for RADC by the Perkin Elmer Company. Descriptions of the device given in their literature (PEI) are generally qualitative. In order for the measurements to be meaningfully compared with theoretical predictions appearing in the literature (Hufnagel, 1963), (Fried, 1966), (Heidbredder, 1967), (Tatarski, 1962) and to obtain a more quantitative understanding of the device operation this analysis was undertaken. Indeed as a result of this work we find that the analytical expression for the quantity measured is not the same as any of those commonly chosen to define angle of arrival. Furthermore the weighting of light entering the input aperture is also found to be quite unique.

Angle of arrival measurements are in general useful because they give angular coordinates of objects. One generally defines angle of arrival as the angles defining the normal to the wavefront of a light wave originating from a point object. If there are no intervening index fluctuations then the wavefront will be a spherical one centered on the object with normals indicating the direction to the object. If there are intervening refractive index fluctuations, then the arriving wavefront may be different from spherical or may be spherical with a different center or radius of curvature, giving a false measurement.

The device senses angle of arrival fluctuations by examining the position of the focused diffraction limited point image. The transverse displacement, δ , is measured and results are interpreted in terms of angle of arrival, α , by the standard relationship $\alpha = \delta/d$ where d is the telescope image distance.

All angle of arrival measurements are limited. Even in the ideal case measurements having better accuracy than the classical angular resolution determined by the input aperture of the optical system, $1.22 \lambda/D$ for a round aperture, are difficult. If there are no refractive index fluctuations the object of a stationary point source is a stationary diffraction pattern the position of the object being determined by the center of the pattern. If there is a spherical wavefront with center modified by the index fluctuations, then the diffraction pattern will be centered at a different spot, the amount of displacement being proportional to the apparent angular deviation of the object. If there is a nonspherical wavefront, then the standard diffraction limited pattern is broken up, the amount of degradation depending on the aperture diameter compared with the mean transverse size of the atmospheric fluctuations. The device determines the mean spot position and averages over the spot shape.

The apparatus is shown schematically in Fig. 1. Light from a point source degraded by the turbulent atmosphere is collected by the receiving telescope and imaged to a degraded diffraction limited spot. The spot is then chopped with the scanning reticle wheel and applied to a detector. The detector output has the form of a periodic signal as shown in Fig. 2. The scanning rate of the wheel is sufficiently fast so that the spot is chopped many times before the atmosphere changes it appreciably. The detector output is then filtered to pass the fundamental frequency. This filtered signal is compared for phase with a standard signal derived from the opposite side of the same scanning wheel. The phase difference is normalized to zero mean, squared and averaged. The mean square phase is then related to the angle of arrival variance in the following analysis.

At some instant of time the voltage output, S , of the signal detector will be given by

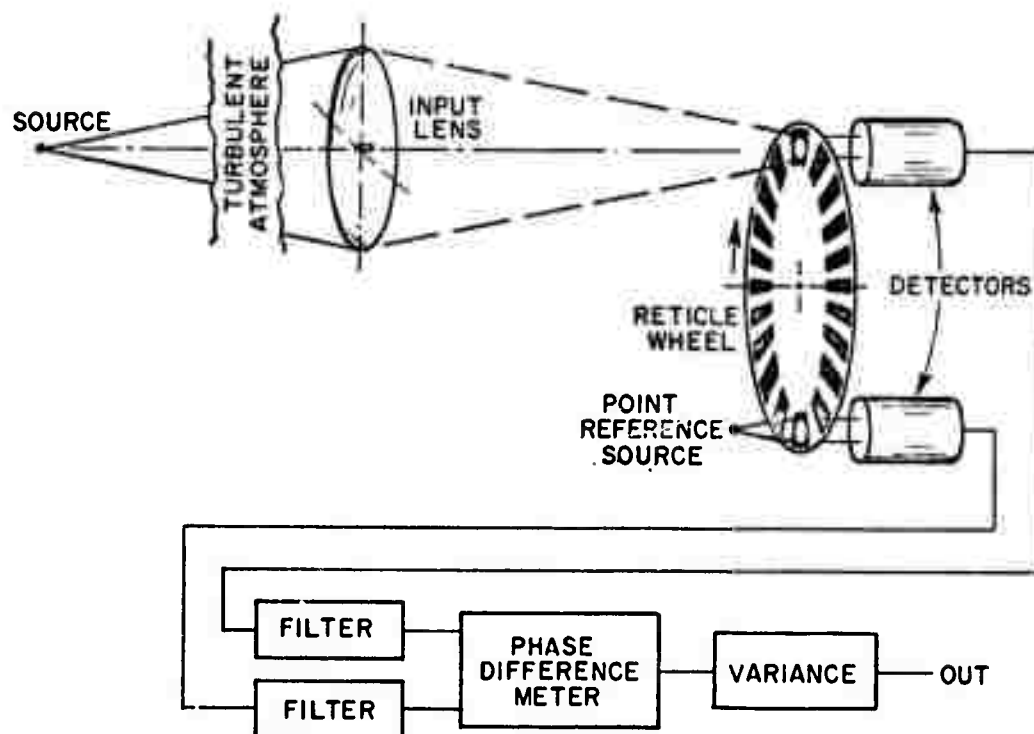


Fig. 1. Angle of arrival apparatus.

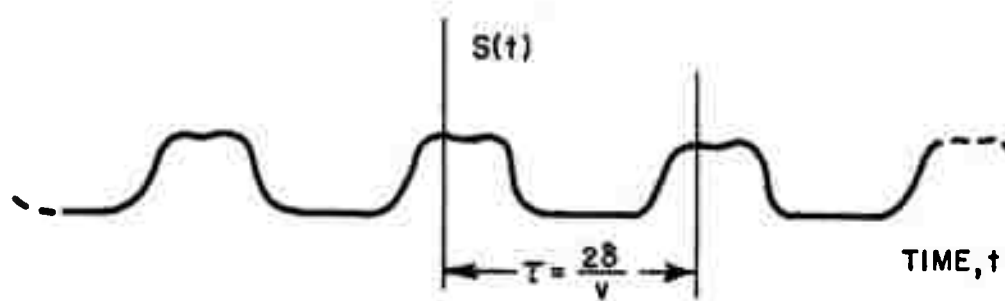


Fig. 2. Typical detector output.

$$(1) \quad S = C_0 \int_{-\infty}^{\infty} dy \int_{x'}^{x''} dx |E_0(\bar{\rho})|^2$$

where $\bar{\rho}$ denotes a point (x,y) in the image plane,

$E_0(\bar{\rho})$ = light field in the image plane

C_0 = constant relating field to output voltage. C_0 contains such factors as detector quantum efficiency, impedance of free space, output impedance, etc.

The slits on the reticle wheel extend sufficiently far in the y direction so as to contain the entire spot. The integration limits x' and x'' denote the x values of the edge of the slit in front of the spot at the instant under consideration.

Using the Fourier transform relationship between the field, $E_1(\bar{r})$, in the telescope aperture plane and the image plane field, the expression for the signal becomes

$$(2) \quad S = C_0 \left(\frac{k}{2\pi d} \right)^2 \int_{-\infty}^{\infty} dy \int_{x'}^{x''} dx \iint_{-\infty}^{\infty} d\bar{r}_1 \iint_{-\infty}^{\infty} d\bar{r}_2 W(\bar{r}_1) W(\bar{r}_2) E_1(\bar{r}_1) E_1^*(\bar{r}_2) e^{j \frac{k}{d} \bar{\rho} \cdot (\bar{r}_1 - \bar{r}_2)}$$

where

\bar{r}_1 and \bar{r}_2 denote positions (r_{1x}, r_{1y}) and (r_{2x}, r_{2y}) in the aperture plane.

$W(\bar{r})$ is the aperture shape function $W=0$ outside the aperture and $W=1$ inside it.

λ = $2\pi/k$ is the light wavelength

d = receiving lens image distance.

The first step is the performance of the x and y integrations. These are straight forward, the results being

$$(3) \quad S = \frac{k C_0}{2\pi d} \iint d\bar{r}_1 \iint d\bar{r}_2 \delta(r_{1y} - r_{2y}) \frac{e^{j \frac{k(r_{1x} - r_{2x})x''}{d}} - e^{j \frac{k(r_{1x} - r_{2x})x'}{d}}}{j \frac{k(r_{1x} - r_{2x})}{d}} \cdot W(\bar{r}_1) W(\bar{r}_2) E(\bar{r}_1) E^*(\bar{r}_2).$$

Equation (3) is put in more tractable form by the transformation to sum and difference coordinates, both in the pupil plane and in the image plane. Thus we define

$$(4) \quad \begin{aligned} \delta &= x'' - x' & \bar{\xi} &= \bar{r}_1 - \bar{r}_2 \\ \sigma &= \frac{x'' + x'}{2} & \bar{\eta} &= \frac{\bar{r}_1 + \bar{r}_2}{2} \end{aligned}$$

δ is the scanning slit width and σ the slit center. At this point it is also appropriate to separate out a flat portion from the wavefront. We define $E_1(\bar{r})$ as the incoming field with the phase measured from a plane normal to the vector κ .

$$(5) \quad E_1(\bar{r}) = E(\bar{r}) e^{-j(\kappa \cdot \bar{r})}$$

The particular choice for κ_x will be indicated later. (The apparatus averages over κ_y .) In terms of these variables, Eq. (3) becomes

$$(6) \quad S(t) = \frac{k C_0 \delta}{2\pi d} \iint d\bar{\xi} \iint d\bar{\eta} \delta(\xi_y) e^{j(\frac{k}{d}\sigma + \kappa_x)\xi_x + j\kappa_y \xi_y} \text{sinc}(\frac{k\xi_x \delta}{2d}) \\ W(\bar{\eta} + \frac{\bar{\xi}}{2}) W(\bar{\eta} - \frac{\bar{\xi}}{2}) E_1(\bar{\eta} + \frac{\bar{\xi}}{2}) E_1^*(\bar{\eta} - \frac{\bar{\xi}}{2}).$$

The $\delta(\xi_y)$ function indicates that the apparatus is insensitive to wavefront tip in the y direction and the spot can have any vertical position in the slit.

We now incorporate the slit motion into the analysis by letting the slit mean position, σ , move linearly in time with constant velocity, v , across the spot.

$$(7) \quad \sigma = vt$$

The slit is assumed to be wider than the spot. We imagine that a humped shaped signal is produced as the slit moves across the spot. This gives one cycle of a periodic pattern that is repeated a large number of times. The wheel velocity is assumed sufficiently fast so that a large number of slits scan the spot before the spot shape changes appreciably, thus generating an essentially periodic function of period

$$(8) \quad \tau = 2\delta/v.$$

The periodic signal will then be represented in the form of a Fourier series. We further adjust the time scale so that the signal peak will come very close to zero time. This is done by adding in a time bias so that the coefficient of ξ_x in Eq. (6) is zero for zero time. To do this we put

$$(9) \quad \frac{k}{d} \sigma - \kappa_x \equiv \frac{k}{d} vt - \kappa_x = \frac{k}{d} vt'.$$

This is equivalent to delaying the time scale so that it coincides with a spot that has been moved sideways by an amount $\kappa_x d/k$ due to the mean plane wavefront tilt component, κ_x .

The form of the Fourier series to be used is that in Eq. (10)

$$(10) \quad S(t) = \sum_{-\infty}^{\infty} a_n e^{j\phi_n} e^{\frac{jn\pi vt}{\delta}} = \sum a_n e^{j\phi_n} e^{\frac{jn\pi v}{\delta} (t' + \frac{\kappa_x d}{kv})}$$

where

$$(11) \quad a_n e^{j\phi_n} = \frac{e}{\tau} e^{j\pi n \frac{\kappa_x d}{k\delta}} \int_{-\delta/v}^{\delta/v} S(t') e^{\frac{jn\pi vt'}{\delta}} dt'.$$

The limits are the natural ones, i.e., those centered at $t'=0$.

Inserting Eq. (7) into Eq. (6), Eq. (6) into Eq. (11) eliminating v and performing the time integration gives

$$(12) \quad a_n e^{j\phi_n} = \frac{k C_0 \delta}{2\pi d} e^{j\pi n \frac{\kappa_x d}{k\delta}} \iint d\bar{\xi} \iint d\bar{\eta} \delta(\xi_y) e^{j\kappa_y \xi_y} \\ \cdot W(\bar{\eta} + \frac{\bar{\xi}}{2}) W(\bar{\eta} - \frac{\bar{\xi}}{2}) E_1(\bar{\eta} + \frac{\bar{\xi}}{2}) E_1^*(\bar{\eta} - \frac{\bar{\xi}}{2}) \text{sinc}(\frac{k\delta \xi_x}{2d}) \text{sinc}(\frac{k\delta}{d} \tau_x - n\pi).$$

We next consider the function performed by the electronic processing equipment. The bandpass filter selects out the signal component at the fundamental frequency. The phase is measured by means of zero point crossings. The phase difference from the reference signal is found and reduced to zero mean, and the mean square phase difference, $\langle (\phi_n - \phi_{ref})^2 \rangle$, is obtained. We want to derive an expression for that mean square phase difference relating it in a simple way to the mean square angle of arrival. The procedure will be to develop an equivalent situation and show how that leads to the desired results.

The first step is to note that the measurement of the zero crossings would be unaffected if a feedback circuit and a variable gain amplifier were added to the system to maintain a constant amplitude for the fundamental harmonic. That would be equivalent to letting the constant C_0 be a random function defined as being proportional to the reciprocal of the amplitude.

We also choose at this time a particular definition of the wave-front normal component, κ_x . Specifically, κ_x is chosen to make the imaginary part of the integral in Eq. (12) zero. That is, denoting the integral in Eq. (12) by I , we have

$$(13) \quad \text{Im } I = \text{Im} \left\{ \iint d\bar{\xi} \iint d\bar{\eta} \delta(\xi_y) e^{j\kappa_y \xi_y} W(\bar{\eta} + \frac{\bar{\xi}}{2}) W(\bar{\eta} - \frac{\bar{\xi}}{2}) \right. \\ \left. \cdot E_1(\bar{\eta} + \frac{\bar{\xi}}{2}) E_1^*(\bar{\eta} - \frac{\bar{\xi}}{2}) \text{sinc}(\frac{\kappa \delta \xi_x}{2d}) \text{sinc}(\frac{\kappa \delta \xi_x}{d} - \pi) \right\} = 0.$$

Taking the magnitude of both sides of Eq. (11) with the imaginary part of the integral equal to zero then gives

$$(14) \quad a_n = \frac{k C_0 \delta}{2\pi d} \text{Re}(I) = \text{constant}.$$

Imagine next that we detect the first harmonic signal giving as output $a_1 e^{j\phi_1}$ and take a long time average of it. Assume that the average is sufficiently long so as to have the ergodic hypothesis hold, and we equate ensemble and time averages. Since the magnitudes are constant, the result would be

$$(15) \quad a_n \langle e^{j\phi_1} \rangle = \langle e^{j\pi \frac{\kappa_x d}{k\delta}} \rangle \frac{k C_0 \delta}{2\pi d} \text{Re}(I).$$

Canceling out the constant factors, assuming a zero mean and normal distribution for the arrival angle fluctuations, and using the relationship $\langle e^{ju} \rangle = e^{1/2 \langle u^2 \rangle}$ for a normally distributed zero mean random variable, u , gives

$$(16) \quad \langle \phi_1^2 \rangle = \frac{\pi d}{k\delta} \langle \kappa_x^2 \rangle.$$

Equation (16) is the desired expression relating the phase variance to the angle of arrival variance. Equation (16) is exactly what one would expect on simple grounds. A given value of κ_x would produce a spot sideways shift of distance $d\kappa_x/k$. This would then correspond to an electronic phase shift of $\phi_1 = (\pi/\delta)(d\kappa_x/k)$, in accord with Eq. (16).

We now want to continue with an examination of the main requirement in obtaining Eq. (16), that the imaginary part of the integral in Eq. (12) be zero, as indicated in Eq. (13). We replace $E_1(\bar{r})$ in terms of its original expression and substitute for the incoming field in terms of a phase and log-amplitude

$$(17) \quad E(\bar{r}) = e^{\ell(\bar{r})} + j\phi(\bar{r}).$$

We further assume that the major part of the phase fluctuations are contained in the wavefront tilt (Fried, 1966) so that the difference between the flat plane and the actual wavefront is less than a tenth radian. The sine of the phase difference $(\phi - \kappa_x x - \kappa_y y)$ can then be replaced by its argument. Equation (13) then simplifies to

$$(18) \quad \kappa_x = N/\Delta$$

where

$$(19) \quad N = \iint d\bar{\xi} \iint d\bar{\eta} \delta(\xi_y) W(\bar{\eta} + \frac{\bar{\xi}}{2}) W(\bar{\eta} - \frac{\bar{\xi}}{2}) \text{sinc}(\frac{k\delta\xi_x}{2d}) \text{sinc}(\frac{k\delta\xi_x}{d} - \pi) \\ e^{\ell(\bar{\eta} + \frac{\bar{\xi}}{2}) + \ell(\bar{\eta} - \frac{\bar{\xi}}{2})} (\phi(\bar{\eta} + \frac{\bar{\xi}}{2}) - \phi(\bar{\eta} - \frac{\bar{\xi}}{2}))$$

$$(20) \quad \Delta = \iint d\bar{\xi} \iint d\bar{\eta} \delta(\xi_y) W(\bar{\eta} + \frac{\bar{\xi}}{2}) W(\bar{\eta} - \frac{\bar{\xi}}{2}) \text{sinc}(\frac{k\delta\xi_x}{2d}) \text{sinc}(\frac{k\delta\xi_x}{d} - \pi) \\ e^{\ell(\bar{\eta} + \frac{\bar{\xi}}{2}) + \ell(\bar{\eta} - \frac{\bar{\xi}}{2})} \xi_x.$$

The expressions in Eqs. (19) and (20) can be simplified. N is the sum of two integrals corresponding to the two ϕ functions in the last parenthesis. We first express Eq. (19) in terms of \bar{r}_1 and \bar{r}_2 using Eq. (4) and then interchange \bar{r}_1 and \bar{r}_2 in the second integral. All the factors remain identical with those in the first integral except for

$$\text{sinc}\left(\frac{k\delta(r_{x1} - r_{x2})}{d} - \pi\right)$$

which goes into

$$\text{sinc}\left(\frac{k\delta(r_{x1} - r_{x2})}{d} + \pi\right).$$

The result is Eq. (21). The expression for Δ in Eq. (20) can be similarly simplified since $\xi_x (= r_{x1} - r_{x2})$ behaves the same as

$$\phi(\bar{r}_1 + \frac{\bar{\xi}}{2}) - \phi(\bar{r}_1 - \frac{\bar{\xi}}{2}) = \phi(\bar{r}_1) - \phi(\bar{r}_2)$$

in Eq. (19) under exchange of \bar{r}_1 and \bar{r}_2 . The result is Eq. (22)

$$(21) \quad N = \iint d\bar{r}_1 W(\bar{r}_1) e^{i\ell(\bar{r}_1)} \phi(\bar{r}_1) F_1(\bar{r}_1)$$

$$(22) \quad \Delta = \iint d\bar{r}_1 W(\bar{r}_1) e^{i\ell(\bar{r}_1)} r_{x1} F_1(\bar{r}_1)$$

where

$$(23) \quad F_1(\bar{r}_1) = \iint d\bar{r}_2 W(\bar{r}_2) \delta(r_{y1} - r_{y2}) e^{i\ell(\bar{r}_2)} \\ \times \text{sinc}\left(\frac{k\delta(r_{x1} - r_{x2})}{2d}\right) \left\{ \text{sinc}\left(\frac{k\delta(r_{x1} - r_{x2})}{d} - \pi\right) - \text{sinc}\left(\frac{k\delta(r_{x1} - r_{x2})}{d} + \pi\right) \right\}$$

The derivation is now restricted to cases where amplitude fluctuations of the incoming beam are negligible. This situation is close to that met for 10.6 micron light at ranges found at the RADC propagation range. With this restriction the coefficient, Δ , of κ_x becomes constant over the ensemble enabling one to write an expression for its mean square value.

$$(24) \quad \langle \kappa_x^2 \rangle = \frac{\langle N^2 \rangle}{\Delta^2}$$

Our final job is now to evaluate expressions for $\langle N^2 \rangle$ and Δ^2 as functions of geometry and coherence parameters. Squaring and averaging the expression for N in Eq. (21), substituting for the correlation function in terms of the relationship

$$(25) \quad \langle \phi(\bar{r}_1) \phi(\bar{r}_1') \rangle = B_\phi(\bar{r}_1 - \bar{r}_1') = \frac{1}{2} (D_\phi(\infty) - D_\phi(\bar{r}_1 - \bar{r}_1'))$$

and recognizing that the integral with $D_\phi(\infty)$ has value zero gives Eq. (26)

$$\begin{aligned}
(26) \quad \langle N^2 \rangle = & -\frac{1}{2} \int d\bar{r}_1 \int d\bar{r}_2 \int d\bar{r}_1' \int d\bar{r}_2' \delta(r_{y1}-r_{y2})^\delta (r_{y1}'-r_{y2}') \\
& \cdot W(\bar{r}_1)W(\bar{r}_2)W(\bar{r}_1')W(\bar{r}_2') \text{sinc}\left(\frac{k\delta}{2d}(r_{x1}-r_{x2})\right) \text{sinc}\left(\frac{k\delta}{2d}(r_{x1}'-r_{x2}')\right) \\
& \{ \text{sinc}\left(\frac{k\delta}{d}(r_{x1}-r_{x2})-\pi\right) - \text{sinc}\left(\frac{k\delta}{d}(r_{x1}-r_{x2})+\pi\right) \} \times \\
& \cdot \{ \text{sinc}\left(\frac{k\delta}{d}(r_{x1}'-r_{x2}')-\pi\right) - \text{sinc}\left(\frac{k\delta}{d}(r_{x1}'-r_{x2}')+\pi\right) \} \times D_\phi(\bar{r}_1-\bar{r}_1').
\end{aligned}$$

Equation (26) can now be partitioned because $D_\phi(\bar{r}_1-\bar{r}_1')$ is independent of \bar{r}_2 and \bar{r}_2' . Separating the \bar{r}_2 and \bar{r}_2' integrations gives

$$(26a) \quad \langle N^2 \rangle = -\frac{1}{2} \int d\bar{r}_1 \int d\bar{r}_1' D_\phi(\bar{r}_1-\bar{r}_1') W(\bar{r}_1)W(\bar{r}_1') F(\bar{r}_1)F(\bar{r}_1')$$

where

$$\begin{aligned}
(26b) \quad F(\bar{r}_1) = & \int d\bar{r}_2 \delta(r_{y1}-r_{y2}) W(\bar{r}_2) \text{sinc}\left(\frac{k\delta}{2d}(r_{x1}-r_{x2})\right) \\
& \cdot \{ \text{sinc}\left(\frac{k\delta}{d}(r_{x1}-r_{x2})-\pi\right) - \text{sinc}\left(\frac{k\delta}{d}(r_{x1}-r_{x2})+\pi\right) \}.
\end{aligned}$$

An expression for Δ^2 can be similarly derived from Eq. (22). It is

$$(26c) \quad \Delta^2 = \int d\bar{r}_1 \int d\bar{r}_1' r_{1x} r_{1x}' W(\bar{r}_1)W(\bar{r}_1') F(\bar{r}_1)F(\bar{r}_1').$$

Equations (25) and (26) will thus be evaluated to give an analytical expression for $\langle \kappa_X^2 \rangle$.

Before proceeding with the numerical expressions for $\langle N^2 \rangle$ and Δ^2 it might be interesting to compare the expression thus obtained for $\langle \kappa_X^2 \rangle$ with comparable expressions appearing in the literature (Fried, 1966), (Heidbreder, 1967). The commonly suggested expression for $\langle \kappa_X^2 \rangle$ is

$$(27) \quad \langle \kappa_X^2 \rangle = \frac{-\frac{1}{2} \int d\bar{r}_1 d\bar{r}_1' D(\bar{r}_1-\bar{r}_1') x_1 x_1' W(\bar{r}_1)W(\bar{r}_1')}{\int d\bar{r}_1 d\bar{r}_1' x_1^2 x_1'^2 W(\bar{r}_1)W(\bar{r}_1')}.$$

The form of the two expressions is indeed identical, the difference being the replacement of the odd function x_1 by another odd function $F(\bar{r}_1)$. $F(\bar{r}_1)$ will be examined further later.

We now proceed with numerical evaluation of $\langle N^2 \rangle$ and Δ^2 . The first step is the nondimensionalization of the relevant expressions with coordinate substitutions, a choice of $D_\phi(\rho)$,

$$(28a) \quad N = k \delta D / 2d$$

$$(28b) \quad u = (r_{x1} - r_{x2}) N / 2D$$

$$(28c) \quad x_1 = 2r_{x1} / D$$

$$(28d) \quad y_1 = 2r_{y1} / D$$

$$(28e) \quad \bar{r}_1 = (x_1, y_1)$$

$$(28f) \quad D_\phi(\rho) = 6.88(\rho/r_0)^{5/3},$$

and determination of the integration limits. The results are

$$(29a) \quad \langle N^2 \rangle = - 6.88 \left(\frac{D}{r_0} \right)^{5/3} \frac{D^4}{2} \int_{-1}^1 dx_1 \int_{-\sqrt{1-x_1^2}}^{+\sqrt{1-x_1^2}} dy_1 \int_{-1}^1 dx'_1 \int_{-\sqrt{1-x_1'^2}}^{+\sqrt{1-x_1'^2}} dy'_1 (\bar{r}_1 - \bar{r}'_1)^{5/3} F(\bar{r}_1) F(\bar{r}'_1)$$

$$(29b) \quad F(\bar{r}_1) = \frac{D}{N} \int_{\frac{N}{2}(x_1 - \sqrt{1-y_1^2})}^{\frac{N}{2}(x_1 + \sqrt{1-y_1^2})} du \operatorname{sinc} u \{ \operatorname{sinc}(2u - \pi) - \operatorname{sinc}(2u + \pi) \}$$

$$(29c) \quad \Delta^2 = + D \int_{-1}^1 dx_1 \int_{-\sqrt{1-x_1^2}}^{+\sqrt{1-x_1^2}} dy_1 \int_{-1}^1 dx'_1 \int_{-\sqrt{1-x_1'^2}}^{+\sqrt{1-x_1'^2}} dy'_1 x_1 x'_1 F(\bar{r}_1) F(\bar{r}'_1).$$

It is interesting to note that the form resulting from the nondimensionalization is precisely that predicted on the basis of very simple arguments multiplied by a correction factor depending on the size of the aperture:

$$(30) \quad \frac{\langle \kappa_x^2 \rangle}{k^2} = \frac{3.44}{k^2 D^2} \left(\frac{D}{r_0} \right)^{5/3} = \frac{D_\phi^2(D)}{2k^2 D^2} .$$

The nondimensional combination $N = k\delta D/2d = \pi D/(\lambda/\delta)d$ is a type of Fresnel number composed of $(\lambda/\delta)d$, that is the diffraction limited length in the aperture plane due to the slit width δ in the image plane. Then $D/(\lambda/\delta)d$ is the number of diffraction limited lengths in the aperture diameter or roughly the number of independent regions across the diameter.

An interesting point about the apparatus can be indicated by examining the function $F(X,Y)$. To do this we first sketch the integrand. This is done in Fig. 3. The limits on the integral in $F(X,Y)$ are symmetric about $NX/2$ and extend a distance $(N/2)\sqrt{1-Y^2}$ in either direction about NX . Examples of various cases for the limits are shown in Fig. 3. In case I in Fig. 3, $NX/2$ is fairly small and $(N/2)\sqrt{1-Y^2}$ is large so that the integration limits include a fair portion of the integrand.

The majority of the area is under the two central lobes so that the integral with limits shown in case I will be quite small, because the positive and negative lobes cancel. However, if the limits are as shown in cases II and III then the integral will be larger because the region of integration includes only one of the central lobes. This case occurs when the semi extent of the integration region, $N/2\sqrt{1-Y^2}$ nearly equals $N/2 X$,

$$(31) \quad \frac{NX}{2} = \frac{N}{2}(1-Y^2)^{1/2} ,$$

or, using Eqs. (28c) and (28d) when

$$r_{x1}^2 + r_{y1}^2 = D^2 .$$

That is $F(X,Y)$ is large when the integrand in the integrals for N and Δ are near the aperture edge. Indeed using the fact that the first zero in the integrand of $F(X,Y)$ occurs for $u=\pi$ it is simple to show that F is large in a ring within a distance roughly $d(\lambda/\delta)$ around the aperture edge. The net result is that light entering the center of the aperture has no effect on the average. Indeed the apparatus might operate more effectively with a better signal-to-noise ratio if an obscure form were inserted to block the aperture center.

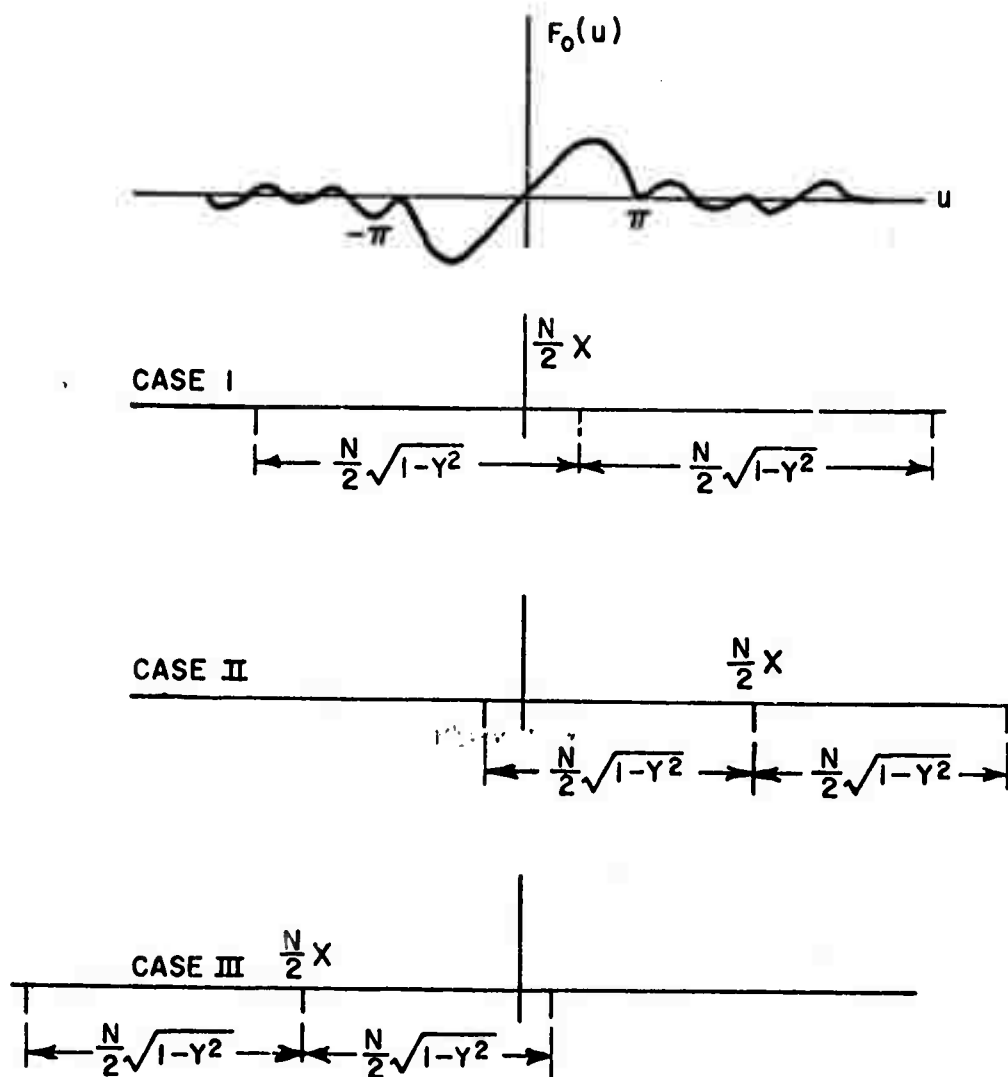


Fig. 3. The integration integrand and three examples of the integration limits.

The integrals in $\langle N^2 \rangle$ and Δ^2 can further be evaluated for very small and very large N . The significant features lie in the limiting expressions for $F_1(\bar{R})$. For example for small N

$$F_1(\bar{R}) = \frac{4}{\pi} \left(\frac{N}{2} \right)^2 X_1 \sqrt{1 - \gamma_1^2}.$$

This occurs in both $\langle N^2 \rangle$ and Δ^2 so that the ratio $\langle N^2 \rangle / \Delta^2$ is independent of N . It can also be shown that for large N $\langle N^2 \rangle / \Delta^2$ is similarly independent of N .

The integrals in $\langle N^2 \rangle$ and Δ^2 have been calculated numerically for the case of large aperture. For $\lambda = 10\mu$, $D = 40$ cm and $\delta = N = 46.103$, $f(N) = .975$. For $N = 92.203$ $f(N) = .971$ illustrating the constancy of $f(N)$ for large N .

To summarize this section, a theoretical description of the particular angle of arrival measuring apparatus has been presented for the case of small amplitude fluctuations. Several conclusions are indicated.

- 1) The device does not measure what has been commonly defined as angle of arrival, but does measure something with similar aperture symmetry properties.
- 2) The basic dependence of arrival angle variance on aperture D , wavelength, $2\pi/k$, and coherence length, r_0 , is that predicted on the basis of simple arguments. It is

$$\langle \alpha^2 \rangle = \frac{D_d(D)}{k^2 D^2}.$$

- 3) For very large and very small apertures, arrival angle variance is independent of $N = k\delta D/2d$ (δ = scanning wheel slot size, d = image distance).
- 4) Only light entering the aperture in a ring of width $d\lambda/\delta$ at the aperture edge is effective in determining the arrival angle variance.

PHASE DIFFERENCE TEMPORAL SPECTRA ALONG SLANT PATHS

The temporal spectrum calculations of phase difference fluctuations for the case when the wind is parallel to the phase point separation have been extended to include the cases of vertical and slant (upward and downward) paths. Input assumptions include turbulence strength C_n^2 is proportional to altitude to the -4/3 power and the outer scale proportional to the altitude. The wind is assumed to be constant along the path. This extends work of previous reports (3432-1).

The calculations have been separated so that the slant path spectrum is obtained from the horizontal path spectrum for a path of the same length and located at the lower end point of the slant path. The additional factor (h) is

$$(1) \quad h = \frac{H_{\max} - H_{\min}}{H_{\min}}$$

where H_{\max} is the upper end point altitude and H_{\min} is the lower end point altitude.

The high, intermediate and low frequency regions of the horizontal path spectrum are given in Table I. Note that for large normalized separation there are only the high and low frequency regions.

TABLE I

PHASE DIFFERENCE SPECTRUM WHEN THE WIND DIRECTION IS PARALLEL TO THE LINE CONNECTING THE PHASE POINTS

$$\Omega = \frac{2\pi f \rho}{v}$$

$$R = \frac{1.071 \rho}{L_o(H_{\min})}$$

$$\underline{\underline{R \ll \sqrt{6}}}$$

$$\frac{W_{\sigma s}(\Omega)}{8.77} \frac{v}{\rho} \left\{ \begin{matrix} 0.5 \\ 1.0 \end{matrix} \right\} k^2 L C_n^2 \rho^{5/3}$$

$$= \frac{1}{6} R^{-8/3} \Omega^2$$

$$R \gg \Omega$$

$$= \frac{1}{6} \Omega^{-2/3}$$

$$\sqrt{6} \gg \Omega \gg R$$

$$= \Omega^{-8/3}$$

$$\Omega \gg \sqrt{6}$$

$$\underline{\underline{R \gg \sqrt{6}}}$$

$$= \frac{1}{6} R^{-8/3} \Omega^2$$

$$6^{3/14} R^{4/7} \gg \Omega$$

$$= \Omega^{-8/3}$$

$$\Omega \gg 6^{3/14} R^{4/7}$$

Figure 4 gives the additional factor to be used when the upward slant path spectral amplitude is desired. It is evident that the low frequencies are accentuated and the intermediate frequencies are attenuated for an upward slant path.

Figure 5 shows the h factor for a downward slant path. Again the low frequencies are accentuated and the intermediate and low frequencies are attenuated.

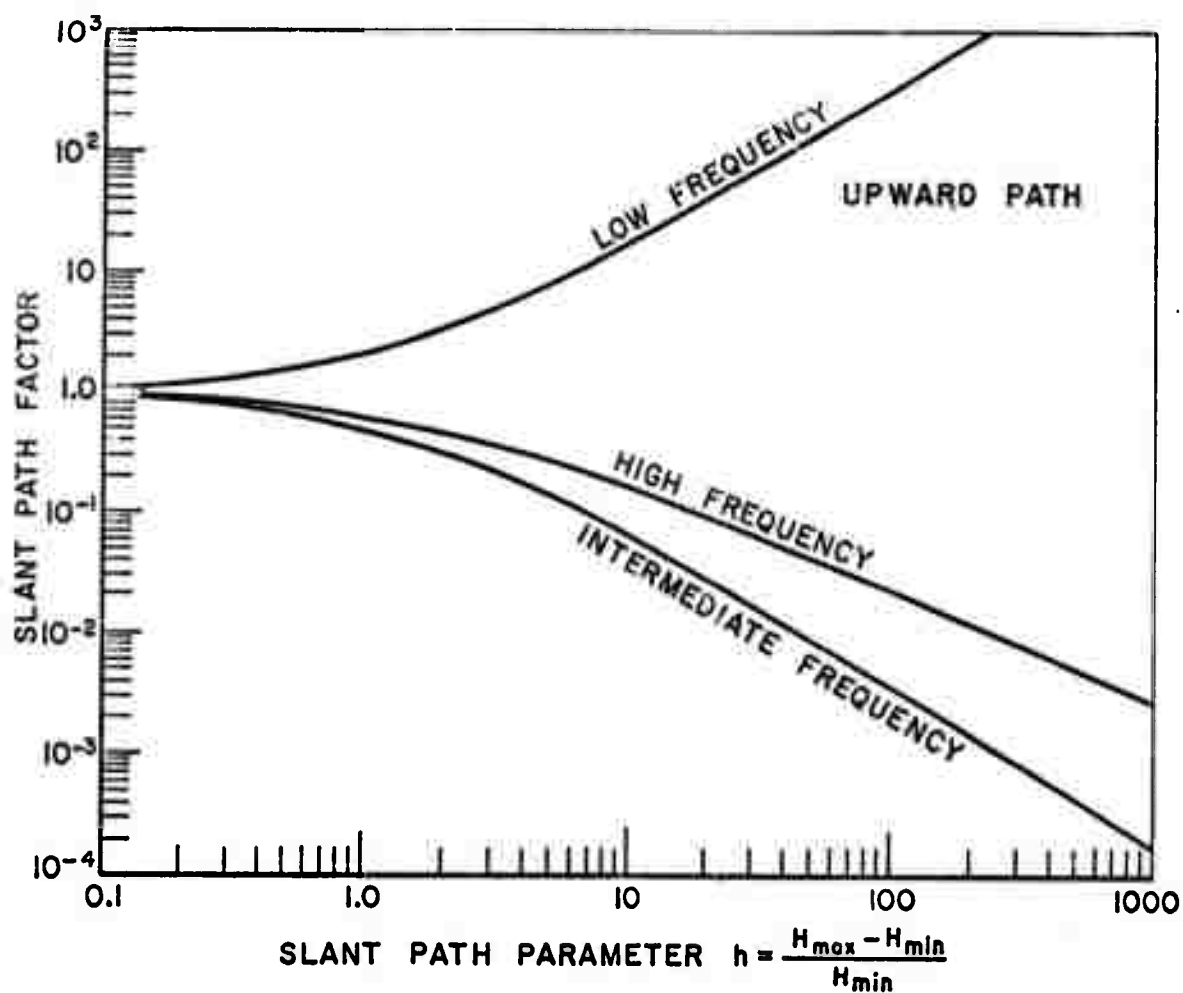


Fig. 4. The additional factor for upward slant path phase difference temporal spectra.

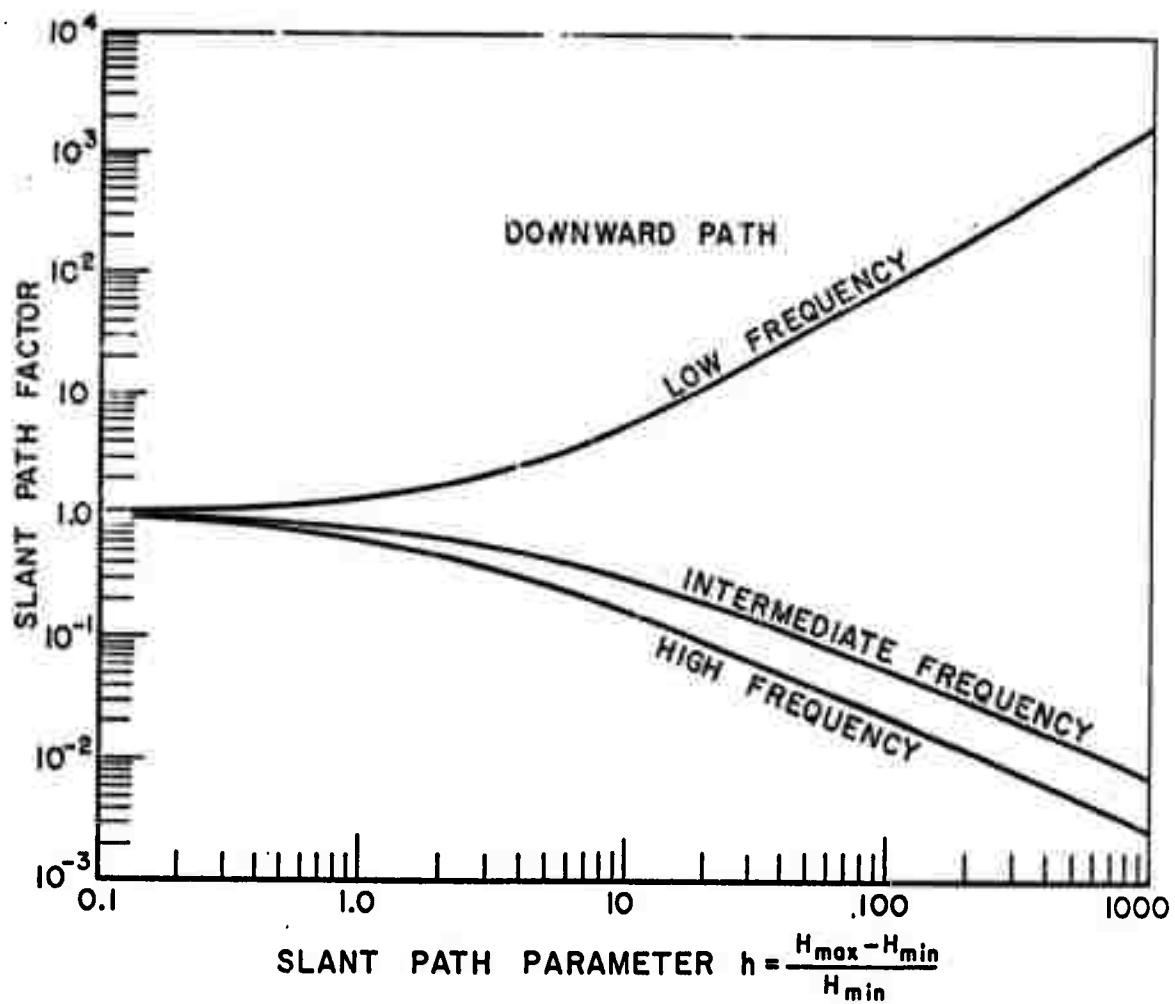


Fig. 5. The additional factor for downward slant path phase difference spectra.

The accentuation of the low frequencies is due to the predominance of outer scale effects at the higher altitudes, and the attenuation of the intermediate and high frequencies is due to the smaller turbulence strength at the higher altitudes.

The horizontal path differential path contribution is essentially constant (3432-2) along the path for high temporal frequencies and hence the upward and downward path high frequency, slant path factors are the same as can be seen by comparing Figs. 1 and 2.

To summarize, temporal phase difference calculations have been extended to include slant and vertical upward and downward paths. The calculations show that spectra are greatly influenced by the variation of outer scale and turbulence strength along the path.

SUMMARY AND CONCLUSIONS

This report covers work performed on Contract F30602-72-C-0305 between July and October, 1972. The work performed is in support of the atmospheric propagation program at RADC. Two items are discussed. The first, a theoretical description of the device used to measure angle of arrival variance at RADC is the conclusion of a project started some time ago. The analysis was performed to obtain a better understanding of the apparatus and its performance. Results show that the device measures a quantity different from, but with symmetry properties similar to, those commonly defined as angle of arrival. Further, only light entering the aperture within a small distance of the rim contributes to the angle of arrival measurement.

The second item is temporal spectra of phase difference fluctuations. This is an extension of previous work and covers slant and vertical paths. The results show that the temporal spectra are strongly influenced by the variation of C_n^2 and L_0 along the path, and that the effects are different for upward and downward paths.

BIBLIOGRAPHY

- (Hufnagel, 1963), Hufnagel, R.E., "Understanding The Physics of Seeing Through Turbulent Atmospheres," Paper presented at Patrick Air Force Base, Florida, (1963).
- (Fried, 1966), Fried, D.L., "Statistics of a Geometric Representation of Wavefront Distortion," J. Opt. Soc. Am., 56, p. 1372, (1966).
- (Heidbreder, 1967), Heidbreder, G.R., "Image Degradation With Random Wavefront Tilt Compensation," IEEE P-GAP, 15, p. 90, (1967).
- (Tatarski, 1967), Tatarski, V.I., Wave Propagation in a Turbulent Atmosphere, Dover, (1967).
- (Collins and Reinhardt, 1972), Collins, S.A., Jr. and Reinhardt, G.W., "Investigation of Laser Propagation Phenomena, Quarterly Report 3432-1, ElectroScience Laboratory, Department of Electrical Engineering, The Ohio State University; prepared under Contract F30602-72-C-0305 for Rome Air Development Center, Griffiss Air Force Base, New York.
- (Reinhardt and Collins, 1972), Reinhardt, G.W. and Collins, S.A., Jr., "Laser Propagation Temporal Spectra," Report 3432-2, ElectroScience Laboratory, Department of Electrical Engineering, The Ohio State University; prepared under Contract F30602-72-C-0305 for Rome Air Development Center, Griffiss Air Force Base, New York. (In preparation).